700+ GMAT Problem Solving Probability and Combinations Questions With Explanations

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Solutions by Bunuel
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1. Mary and Joe are to throw three dice each. The score is the sum of points on all three dice. If Mary scores 10 in her attempt what is the probability that Joe will outscore Mary in his?

A. 24/64

B. 32/64

C. 36/64

D. 40/64

E. 42/64

Expected value of a roll of one dice is 1/6(1+2+3+4+5+6)=3.5.
Expected value of three dices is 3*3.5=10.5.

Mary scored 10 so the probability to have more then 10, or more then average is the same as to have less than average=1/2.

P=1/2.

Answer: B.

Discussed at: http://gmatclub.com/forum/mother-mary-comes-to-me-86407.html

2. Denise is trying to open a safe whose combination she does not know. IF the safe has 4000 possible combinations, and she can try 75 different possibilities, what is the probability that she does not pick the one correct combination.
A. 1
B. 159/160
C. 157/160
D. 3/160
E. 0

When trying the first time the probability Denise doesn't pick the correct combination=3999/4000
Second time, as the total number of possible combinations reduced by one, not picking the right one would be 3998/3999.
Third time 3997/3998
...
And the same 75 times.

So we get: $3999/4000*3998/3999*...*3925/3926$ every denominator but the first will cancel out and every nominator but the last will cancel out as well.

We'll get 3925/4000=157/160.

Answer: C.

Discussed at: http://gmatclub.com/forum/4000-possible-combination-84435.html

3. A box contains 10 pairs of shoes (20 shoes in total). If two shoes are selected at random, what it is the probability that they are matching shoes?
A. 1/190
B. 1/20
C. 1/19

D. 1/10
E. 1/9

The probability would simply be: 1/1*1/19(as after taking one at random there are 19 shoes left and only one is the pair of the first one)=1/19

Answer: C.

We can solve it in another way:

P= Favourable outcomes/Total # of outcomes

Favourable outcomes = 10C1 as there are 10 pairs and we need ONE from these 10 pairs.
Total # of outcomes= 20C2 as there are 20 shoes and we are taking 2 from them.

P=10C1/20C2 =10/(19*10)=1/19

Answer: C.

Discussed at: http://gmatclub.com/forum/probability-that-they-are-matching-shoes-85916.html

4. A Coach is filling out the starting lineup for his indoor soccer team. There are 10 boys on the team, and he must assign 6 starters to the following positions: 1 goalkeeper, 2 on defence, 2 in midfield, and 1 forward. Only 2 of the boys can play goalkeeper, and they cannot play any other positions. The other boys can each play any of the other positions. How many different groupings are possible?
A. 60
B. 210
C. 2580
D. 3360
E. 151200

2C1 select 1 goalkeeper from 2 boys;
8C2 select 2 defence from 8 boys (as 2 boys can only play goalkeeper 10-2=8);
6C2 select 2 midfield from 6 boys (as 2 boys can only play goalkeeper and 2 we've already selected for defence 10-2-2=6);
4C1 select 1 forward from 4 boys (again as 2 boys can play only goalkeeper, 4 we've already selected for defence and midfield 10-2-4=4)

Total # of selection=2C1*8C2*6C2*4C1=3360

Answer: D.

Discussed at: http://gmatclub.com/forum/combination-or-permutation-can-t-make-up-my-mind-85800.html

5. In how many ways 8 different tickets can be distributed between Jane and Bill if each is to receive any even number of tickets and all 8 tickets to be distributed.
A. From 2 to 6 inclusive.
B. From 98 to 102 inclusive.
C. From 122 to 126 inclusive.
D. From 128 to 132 inclusive.
E. From 196 to 200 inclusive.

Tickets can be distributed in the following ways:

{8,0} - 8C8=1
{6,2} - 8C6*2C2=28
{4,4} - 8C4*4C4=70
{2,6} - 8C2*6C6=28
{0,8} - 8C8=1

Total # of ways=1+28+70+28+1=128

Answer: D.

Discussed at: http://gmatclub.com/forum/sharing-tickets-87128.html

6. An insect has one shoe and one sock for each of its twelve legs. In how many different orders can the insect put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

A. $2^{12}*12!$

B. $\frac{24!}{12!^2}$

C. $\frac{24!}{2^{12}}$

D. $24!$

E. $24!*12^2$

Nothing like this will **ever** occur at real test, as this question is beyond the scope of GMAT. So this question is just for practice.

NOTE that each sock and shoe is "assigned" to a specific leg.

Imagine situation with no restriction, meaning no need to put the socks before the shoes. In this case the # of ways insect can put 24 items would be 24!. As we can choose to put ANY of 24 items first, then 23 items left, then 22 and so on.

Next step. On EACH leg we can put either sock OR shoe first. But for EACH leg from 12, only one order is correct WITH restriction: sock first then shoe. For one leg chances of correct order is 1/2, for two legs 1/2^2, similarly for 12 legs chances of correct order would 1/2^12.

So we get that for the total # of ways, WITH NO RESTRICTION, which is 24!, only 1/2^12 is good WITH RESTRICTION.

So the final answer is 24!/2^12.

Answer: C.

Discussed at: http://gmatclub.com/forum/insect-87503.html

7. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually six is
A. 1/8
B. 2/8
C. 3/8
D. 1/2

P=Favorable outcomes/Total # of possible outcomes.

Favorable outcome is that it's actually six and he's telling the truth $=\frac{3}{4}*\frac{1}{6}=\frac{1}{8}$

Total # of possible outcomes is: either it's six and he's telling the truth OR it's not six and he's telling the lie $=\frac{3}{4}*\frac{1}{6}+\frac{1}{4}*\frac{5}{6}=\frac{1}{3}$

$$P=\frac{\frac{1}{8}}{\frac{1}{3}}=\frac{3}{8}$$

Discussed at: http://gmatclub.com/forum/six-or-not-six-87984.html

8. In how many ways can 11 books on English and 9 books on French be placed in a row on a shelf so that two books on French may not be together?

We have 11 English and 9 French books, no French books should be adjacent.

Imagine 11 English books in a row and empty slots like below:

*E*E*E*E*E*E*E*E*E*E*E*

Now if 9 French books would be placed in 12 empty slots, all French books will be separated by English books.

So we can "choose" 9 empty slots from 12 available for French books, which is 12C9=220.

Answer: 220.

Discussed at: http://gmatclub.com/forum/permutation-combination-bookshelf-87352.html

9. In the xy-plane, the vertex of a square are (1, 1), (1,-1), (-1, -1), and (-1,1). If a point falls into the square region, what is the probability that the ordinates of the point (x,y) satisfy that x^2+y^2>1?
A. 1-pi/4
B. pi/2
C. 4-pi
D. 2-pi
E. pi-2

First note that the square we have is centered at the origin, has the length of the sides equal to 2 and the area equal to 4.

$x^2+y^2=1$ is an equation of a circle also centered at the origin, with radius 1 and the $area=\pi r^2=\pi$.

We are told that the point is IN the square and want to calculate the probability that it's outside the circle ($x^+y^2>1$ means that the point is outside the given circle).

P=Favorable outcome/Total number of possible outcomes.

Favorable outcome is the area between the circle and the square= $4-\pi$
Total number of possible outcomes is the area of the square (as given that the point is in the square) =4

$$P=\frac{4-\pi}{4}=1-\frac{\pi}{4}$$

Answer: A.

Discussed at: http://gmatclub.com/forum/probability-88246.html

10. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if 4 letters are used at a time?

A. 360
B. 720
C. 240
D. 120
E. 60

Choosing 4 letters out of 6 (distinct) letters to form the word = 6C4=15;
Permutations of these 4 letters = 4!=24;

Total # of words possible = 15*24= 360

Answer: A.

Discussed at: http://gmatclub.com/forum/permutation-question-88492.html

11. In how many ways can the letters of the word PERMUTATIONS be arranged if there are always 4 letters between P and S

A. 2419200
B. 25401600
C. 1814400
D. 1926300
E. 1321500

There are 12 letters in the word "PERMUTATIONS", out of which T is repeated twice.

1. Choosing 4 letters out of 10 (12-2(P and S)=10) to place between P and S = 10C4 = 210;
2. Permutation of the letters P ans S (PXXXXS or SXXXXP) = 2! =2;
3. Permutation of the 4 letters between P and S = 4! =24;
4. Permutations of the 7 units {P(S)XXXXS(P)}{X}{X}{X}{X}{X}{X} = 7! = 5040;
5. We should divide multiplication of the above 4 numbers by 2! as there is repeated T.

Hence: $\frac{10C4*2!*4!*7!}{2!}=25{,}401{,}600$

Answer: B.

Discussed at: http://gmatclub.com/forum/permutation-question-88492.html

12. 4 dices are thrown at the same time. whats the probability of getting ONLY 2 dices showing the same face?

I suppose "only 2 dice showing the same face" means EXACTLY two? If so then:

Total # of outcomes = 6^4

Favorable outcomes = 4C2=6, choosing two dice which will provide the same face, these two dice can take 6 values, other two 5 and 4. So, favorable outcomes=4C2*6*5*4.

$$P=\frac{4C2*6*5*4}{6^4}=\frac{5}{9}$$

Answer: 5/9.

Discussed at: http://gmatclub.com/forum/really-tough-88487.html

13. Right triangle ABC is to be drawn in the xy-plane so that the right angle is at A and AB is parallel to the y-axis. If the x- and y-coordinates of A, B, and C are to be integers that are consistent with the inequalities -6 ≤ x ≤ 2 and 4 ≤ y ≤ 9 , then how many different triangles can be drawn that will meet these conditions?
A. 54
B. 432
C. 2160
D. 2916
E. 148,824

We have the rectangle with dimensions 9*6 (9 horizontal dots and 6 vertical). AB is parallel to y-axis and AC is parallel to x-axis.

Choose the (x,y) coordinates for vertex A: 9C1*6C1;
Choose the x coordinate for vertex C (as y coordinate is fixed by A): 8C1, (9-1=8 as 1 horizontal dot is already occupied by A);
Choose the y coordinate for vertex B (as x coordinate is fixed by A): 5C1, (6-1=5 as 1 vertical dot is already occupied by A).

9C1*6C*8C1*5C1=2160.

Answer: C.

Discussed at: http://gmatclub.com/forum/tough-problem-88958.html

14. In a certain game, a large bag is filled with blue, green, purple and red chips worth 1, 5, x and 11 points each, respectively. The purple chips are worth more than the green chips, but less than the red chips. A certain number of chips are then selected from the bag. If the product of the point values of the selected chips is 88,000, how many purple chips were selected?
A. 1
B. 2
C. 3
D. 4
E. 5

$88,000 = 2^6 * 5^3 * 11$, as no chip's value is multiple of 2, hence 2^6=64 must be the product of the values of the purple chips drawn. The value of the purple chip is multiple of 2, but more than 5 and less than 11, hence it's 8 (2^3). 8*8=64, two purple chips were drawn.

Answer: B (2).

Discussed at: http://gmatclub.com/forum/chips-worth-points-89694.html

15. In how many different ways can trhee letters be posted from seven different postboxes assuming no two letters can be posted from the same postbox?

First letter could be sent from ANY of the seven postboxes - 7 (7 options);
Second letter could be sent from the SIX postboxes left - 6 (6 options);
Third letter could be sent from the FIVE postboxes left - 5 (5 options);

Total # of ways =7*6*5=210

What if there is no restriction, that is, if two or more letters can be posted from the same box?

In this problem we don't have restriction, thus ANY letter could be sent from ANY postboxes =7*7*7=7^3=343

Discussed at: http://gmatclub.com/forum/counting-principles-55340.html

16. Possible arrangements for the word REVIEW if one E can't be next to the other.

THEORY:

Permutations of n things of which P1 are alike of one kind, P2 are alike of second kind, P3 are alike of third kind .... Pr are alike of r th kind such that: P1+P2+P3+..+Pr=n is:

$$\frac{n!}{P1!*P2!*P3!*..*Pr!}.$$

For example number of permutation of the letters of the word "gmatclub" is 8! as there are 8 DISTINCT letters in this word.

Number of permutation of the letters of the word "google" is 6!/2!2!, as there are 6 letters out of which "g" and "o" are represented twice.

Number of permutation of 9 balls out of which 4 are red, 3 green and 2 blue, would be 9!/4!3!2!.

In the original question there are 6 letters out of which E appears twice. Total number of permutation of these letters (without restriction) would be: $\frac{6!}{2!}=360$.

# of combination for which two E **are adjacent** is $5!=120$, (consider two E as one element like: {R}{EE}{V}{I}{W}: # of permutation of this 5 elements is $5!=120$)

Total # of permutation for which two E **are not adjacent** would be $360-120=240$.

Discussed at: http://gmatclub.com/forum/premutations-and-combinations-90174.html

17. In how many different ways can the letters A,A,B,B,B,C,D,E be arranged if the letter C must be to the right of the letter D?
A. 1680
B. 2160
C. 2520
D. 3240
E. 3360

We have 8 letters out of which A appears twice and B appears three time. Total number of permutation of these letters (without restriction) would be: $\frac{8!}{2!3!}=3360$.

Now, in half of these cases D will be to the right of C and in half of these cases to the left, hence the final answer would be $\frac{3360}{2}=1680$

Answer: A.

Discussed at: http://gmatclub.com/forum/probability-q-91460.html

18. If there are 85 students in a statistics class and we assume that there are 365 days in a year, what is the probability that at least two students in the class have the same birthday (assuming birthdays are distributed independently)?

A. (85/365)* (84/364)
B. (1/365)* (1/364)
C. 1- (85!/365!)
D. 1- (365!/ 280! (365^85))
E. 1- (85!/(365^85))

The easiest way to solve this problem is to calculate opposite probability and subtract this value from 1:

The opposite probability is that all students have the birthdays on different days: $\frac{365}{365}*\frac{364}{365}*\frac{363}{365}*...*\frac{281}{365}=\frac{365*364*363*...*281}{365^{85}}=\frac{365!}{280!*365^{85}}$ total 85 birthdays (first student can have birthday on any day =1=365/365, the probability that the second student will have the birthday on another day is 364/365, the probability that the third student will have the birthday not on this two days is 363/365, and so on).

So, the probability that at least two students in the class have the same birthday is: $1-\frac{365!}{280!*365^{85}}$.

Answer: D.

Discussed at: http://gmatclub.com/forum/probability-700-difficulty-level-92013.html

19. How many words can be formed by taking 4 letters at a time out of the letters of the word MATHEMATICS.

There are 8 distinct letters: M-A-T-H-E-I-C-S. 3 letters M, A, and T are represented twice (double letter).

Selected 4 letters can have following 3 patterns:

1. abcd - all 4 letters are different:
$8P4=1680$ (choosing 4 distinct letters out of 8, when order matters) or $8C4*4!=1680$ (choosing 4 distinct letters out of 8 when order does not matter and multiplying by 4! to get different arrangement of these 4 distinct letters);

2. aabb - from 4 letters 2 are the same and other 2 are also the same:
$3C2*\frac{4!}{2!2!}=18$ - 3C2 choosing which two double letter will provide two letters (out of 3 double letter - MAT), multiplying by $\frac{4!}{2!2!}$ to get different arrangements (for example MMAA can be arranged in $\frac{4!}{2!2!}$ # of ways);

3. aabc - from 4 letters 2 are the same and other 2 are different:
$3C1*7C2*\frac{4!}{2!}=756$ - 3C1 choosing which letter will proved with 2 letters (out of 3 double letter - MAT), 7C2 choosing third and fourth letters out of 7 distinct letters left and multiplying by $\frac{4!}{2!}$ to get different arrangements (for example MMIC can be arranged in $\frac{4!}{2!}$ # of ways).

1680+18+756=2454

Answer: 2454.

Discussed at: http://gmatclub.com/forum/tough-p-n-c-92675.html

20. In how many different ways can 4 physics, 2 math and 3 chemistry books be arranged in a row so that all books of the same branch are together?
A. 1242
B. 1728
C. 1484
D. 1734
E. 1726

There are three branches, three units of books: {physics}{math}{chemistry} - aranging branches 3!;

Arranging the books within the branches:
physics - 4!;
math - 2!;
chemistry - 3!;

Total: 3!*4!*2!*3!.

Answer: B.

Discussed at: http://gmatclub.com/forum/perm-55383.html

21. In how many ways can the letters of the word PERMUTATIONS be arranged if there are always 4 letters between P and S?

There are 12 letters in the word "PERMUTATIONS", out of which T is repeated twice.

1. Choosing 4 letters out of 10 (12-2(P and S)=10) to place between P and S = 10C4 = 210;
2. Permutation of the letters P ans S (PXXXXS or SXXXXP) = 2! =2;
3. Permutation of the 4 letters between P and S = 4! =24;
4. Permutations of the 7 units {P(S)XXXXS(P)}{X}{X}{X}{X}{X}{X} = 7! = 5040;
5. We should divide multiplication of the above 4 numbers by 2! as there is repeated T.

Hence: $\frac{10C4*2!*4!*7!}{2!}=25,401,600$

Discussed at: http://gmatclub.com/forum/interesting-problems-of-permutations-and-combinations-94381.html

22. In how many of the distinct permutations of the letters in the word MISSISSIPPI do the 4 I's not come together?

There are 11 letters in the word "MISSISSIPPI ", out of which: M=1, I=4, S=4, P=2.

Total # of permutations is $\frac{11!}{4!4!2!}$;

\# of permutations with 4 I's together is $\frac{8!}{4!2!}$. Consider 4 I's as one unit: {M}{S}{S}{S}{S}{P}{P}{IIII} - total 8 units, out of which {M}=1, {S}=4, {P}=2, {IIII}=1.

So # of permutations with 4 I's **not come together** is: $\frac{11!}{4!4!2!}-\frac{8!}{4!2!}$.

Discussed at: http://gmatclub.com/forum/interesting-problems-of-permutations-and-combinations-94381.html

22. A ...firm is divided into four departments, each of which contains four people. If a project is to be assigned to a team of three people, none of which can be from the same department, what is the greatest number of distinct teams to which the project could be assigned?
A 4^3
B. 4^4
C. 4^5
D. 6(4^4)
E. 4(3^6)

C_4^3 # of ways to choose which 3 department will provide employee for the team and as each chosen department can provide with 4 employees then total # of different teams will be $C_4^3*4*4*4=4^4$.

Discussed at: http://gmatclub.com/forum/combinations-problem-extreme-challenge-a-firm-is-84160.html

23. How many positive integers less than 10,000 are there in which the sum of the digits equals 5?
A. 31
B. 51
C. 56
D. 62
E. 93

Consider this: we have 5 d's and 3 separators $|$, like: $ddddd|||$. How many permutations (arrangements) of these symbols are possible? Total of 8 symbols (5+3=8), out of which 5 d's and 3 $|$'s are identical, so $\frac{8!}{5!3!}=56$.

With these permutations we'll get combinations like: $|dd|d|dd$ this would be 3 digit number 212 OR $|||ddddd$ this would be single digit number 5 (smallest number less than 10,000 in which sum of digits equals 5) OR $ddddd|||$ this would be 4 digit number 5,000 (largest number less than 10,000 in which sum of digits equals 5)...

Basically this arrangements will give us all numbers less than 10,000 in which sum of the digits (sum of 5 d's=5) equals 5.

Hence the answer is $\frac{8!}{5!3!}=56$.

Answer: C (56).

This can be done with direct formula as well:

The total number of ways of dividing n identical items (5 d's in our case) among r persons or objects (4 digt places in our case), each one of whom, can receive**0, 1, 2** or more items (from zero to 5 in our case) is ${}^{n+r-1}C_{r-1}$.

In our case we'll get: ${}^{n+r-1}C_{r-1}={}^{5+4-1}C_{4-1}=8C3=\frac{8!}{5!3!}=56$

Discussed at: http://gmatclub.com/forum/integers-less-than-85291.html

24. A local bank that has 15 branches uses a two-digit code to represent each of its branches. The same integer can be used for both digits of a code, and a pair of two-digit numbers that are the reverse of each other (such as 17 and 71) are considered as two separate codes. What is the fewest number of different integers required for the 15 codes?

A. 3
B. 4
C. 5
D. 6
E. 7

Consider the code XY. If there are n digits available then X can take n values and Y can also take n values, thus from n digits we can form n^2 different 2-digit codes: this is the same as from 10 digits (0, 1, 2, 3, ..., 9) we can form 10^2=100 different 2-digit numbers (00, 01, 02, ..., 99).

We want # of codes possible from n digit to be at least 15 --> $n^2 \geq 15$ --> $n \geq 4$, hence min 4 digits are required.

Answer: B.

Discussed at: http://gmatclub.com/forum/permutation-question-98109.html

25. Mrs. Smith has been given film vouchers. Each voucher allows the holder to see a film without charge. She decides to distribute them among her four nephews so that each nephew gets at least two vouchers. How many vouchers has Mrs. Smith been given if there are 120 ways that she could distribute the vouchers?
A. 13
B. 14
C. 15
D. 16
E. more than 16

Clearly there are more than 8 vouchers as each of four can get at least 2. So, basically 120 ways vouchers can the distributed are the ways to distribute $x-8$vouchers, so that each can get from zero to $x-8$ as at "least 2", or 2*4=8, we already booked. Let $x-8$ be k.

In how many ways we can distribute k identical things among 4 persons? Well there is a formula for this but it's better to understand the concept.

Let $k=5$. And imagine we want to distribute 5 vouchers among 4 persons and each can get from zero to 5, (no restrictions).

Consider:

$ttttt|||$
We have 5 tickets (t) and 3 separators between them, to indicate who will get the tickets:

$ttttt|||$
Means that first nephew will get all the tickets,

$|t|ttt|t$
Means that first got 0, second 1, third 3, and fourth 1

And so on.

How many permutations (arrangements) of these symbols are possible? Total of 8 symbols (5+3=8), out of which 5 t's and 3 $|$'s are identical, so $\frac{8!}{5!3!}=56$. Basically it's the number of ways we can pick 3 separators out of 5+3=8: $8C3$.

So, # of ways to distribute 5 tickets among 4 people is $(5+4-1)C(4-1)=8C3$.

For k it will be the same: # of ways to distribute k tickets among 4 persons (so that each can get from zero to k) would be $(K+4-1)C(4-1)=(k+3)C3=\frac{(k+3)!}{k!3!}=120$.

$(k+1)(k+2)(k+3)=3!*120=720$. --> $k=7$. Plus the 8 tickets we booked earlier: $x=k+8=7+8=15$.

Answer: C (15).

P.S. How to solve $(k+1)(k+2)(k+3)=3!*120=720$: 720, three digit integer ending with 0, is the product of three consecutive integers. Obviously one of them must be multiple of 5: try $5*6*7=210<720$, next possible triplet $8*9*10=720$, OK. So $k+1=8$ --> $k=7$.

P.P.S. Direct formula:

The total number of ways of dividing n identical items among r persons, each one of whom, can receive 0,1,2 or more items is $^{n+r-1}C_{r-1}$**.**

The total number of ways of dividing n identical items among r persons, each one of whom receives at least one item is $^{n-1}C_{r-1}$**.**

Discussed at: http://gmatclub.com/forum/voucher-98225.html

26. In the Mundane Goblet competition, 6 teams compete in a "round robin" format: that is, each team plays every other team exactly once. A team gets 3 points for a win, 1 point for a tie (a draw), and 0 points for a loss. What is the difference between the maximum total points and the minimum total points that can be gained by all teams (added together) in the Mundane Goblet competition?
A. 15
B. 30
C. 45
D. 60
E. 75

There will be $C^2_6=15$ games needed so that each team to play every other team exactly once ($C^2_6=15$ is the # of ways we can pick two different teams to play each other).

Now, in one game max points (3 points) will be obtained if one team wins and another looses and min points (1+1=2 points) will be obtained if there will be a tie. Hence, maximum points that can be gained by all teams will be **15 games * 3 points=45** and the minimum points that can be gained by all teams will be **15 games * 2 points=30**, difference is **45-30=15**.

Answer: A.

Discussed at: http://gmatclub.com/forum/mbamission-the-quest-for-96952.html

26. Seven men and five women have to sit around a circular table so that no 2 women are together. In how many different ways can this be done?

# of arrangements of 7 men around a table is $(7-1)!=6!$;
There will be 7 possible places for women between them, 7 empty slots. # of ways to choose in which 5 slots women will be placed is $C^5_7=21$;
# of arrangements of 5 women in these slots is $5!$;

So total: $6!*21*5!=1,814,400$.

Answer: 1,814,400.

Discussed at: http://gmatclub.com/forum/arrangement-in-a-circle-98185.html

27. If x is a randomly chosen integer between 1 and 20, inclusive, and y is a randomly chosen integer between 21 and 40, inclusive, what is the probability that xy is a multiple of 4?
A. 1/4
B. 1/3
C. 3/8
D. 7/16
E. ½

Let's find the probability of an opposite event and subtract this value from 1.

There are three cases xy NOT to be a multiple of 4:
A. both x and y are odd --> 1/2*1/2=1/4;
B. x is odd and y is even but not multiple of 4 --> 1/2*1/4=1/8;
C. y is odd and x is even but not multiple of 4 --> 1/2*1/4=1/8.

$$P(xy=multiple\ of\ 4)=1-(\frac{1}{4}+\frac{1}{8}+\frac{1}{8})=\frac{1}{2}$$

Answer: E.

Discussed at: http://gmatclub.com/forum/hard-one-98842.html

28. A password to a certain database consists of digits that cannot be repeated.If the password is known to consist of at least 8 digits and it takes 12 seconds to try one combination , what is the amount of time , in minutes , necessary to guarantee access to database?

A. 8!/5

B. 8!/2

C. 8!

D. 10!/2

E. 5/2.10!

Password can have 8, 9 or 10 digits (more than 10 is not possible as per stem digits must be distinct).

Total # of passwords possible for 8 digits is $P^8_{10}=\frac{10!}{2}$;

Total # of passwords possible for 9 digits is $P^9_{10}=10!$;

Total # of passwords possible for 10 digits is $P^{10}_{10}=10!$.

Time needed to guarantee access to database is $(\frac{10!}{2}+10!+10!)*\frac{1}{5}=\frac{10!}{2}$ minutes.

Answer: D.

Discussed at: http://gmatclub.com/forum/permutation-and-combination-95496.html

29. A row of seats in a movie hall contains 10 seats. 3 Girls & 7 boys need to occupy those seats. What is the probability that no two girls will sit together?

Consider the following:
*B*B*B*B*B*B*B*

Now, if girls will occupy the places of 8 stars no girl will sit together.

# of ways 3 girls can occupy the places of these 8 stars is C^3_8;
# of ways 3 girls can be arranged on these places is $3!$;
# of ways 7 boys can be arranged is $7!$.

So total # of ways to arrange 3 Girls and 7 boys so that no girls are together is $C^3_8*3!*7!$;
Total # of ways to arrange 10 children is $10!$.

So $P=\frac{C^3_8*3!*7!}{10!}=\frac{7}{15}$.

Discussed at: http://gmatclub.com/forum/ps-probability-3girls-7-boys-99268.html

30. A fair coin is tossed 5 times. What is the probability of getting at least three heads on consecutive tosses?
A 2/16
B 1/4
C 7/24
D 5/16
E 15/32

At least 3 heads means 3, 4, or 5 heads.

3 *consecutive* heads
5 cases:
HHHTT
THHHT
TTHHH
HTHHH
HHHTH

$P=5*(\frac{1}{2})^5=\frac{5}{32}$.

4 *consecutive* heads
2 cases:
HHHHT
THHHH

$P=2*(\frac{1}{2})^5=\frac{2}{32}$.

5 *consecutive* heads
1 case:
HHHHH

$P=(\frac{1}{2})^5=\frac{1}{32}$.

$P=\frac{5}{32}+\frac{2}{32}+\frac{1}{32}=\frac{8}{32}=\frac{1}{4}$.

Answer: B.

Discussed at: http://gmatclub.com/forum/hard-probability-99478.html

31. Bill has a small deck of 12 playing cards made up of only 2 suits of 6 cards each. Each of the 6 cards within a suit has a different value from 1 to 6; thus, for each value from 1 to 6, there are two cards in the deck with that value. Bill likes to play a game in which he shuffles the deck, turns over 4 cards, and looks for pairs of cards that have the same value. What is the chance that Bill finds at least one pair of cards that have the same value?
A. 8/33
B. 62/165
C. 17/33
D. 103/165
E. 25/33

Let's calculate the opposite probability ans subtract this value from 1.

Opposite probability would be that there will be no pair in 4 cards, meaning that all 4 cards will be different: $\frac{C_6^4*2^4}{C_{12}^4}=\frac{16}{33}$.

C_6^4 - # of ways to choose 4 different cards out of 6 different values;
2^4 - as each of 4 cards chosen can be of 2 different suits;
C_{12}^4 - total # of ways to choose 4 cards out of 12.

So $P=1-\frac{16}{33}=\frac{17}{33}$.

Or another way:

We can choose any card for the first one - $\frac{12}{12}$;

Next card can be any card but 1 of the value we'v already chosen - $\frac{10}{11}$ (if we've picked 3, then there are one more 3 left and we can choose any but this one card out of 11 cards left);

Next card can be any card but 2 of the values we'v already chosen - $\frac{8}{10}$ (if we've picked 3 and 5, then there are one 3 and one 5 left and we can choose any but these 2 cards out of 10 cards left);

Last card can be any card but 3 of the value we'v already chosen - $\frac{6}{9}$;

$P=\frac{12}{12}*\frac{10}{11}*\frac{8}{10}*\frac{6}{9}=\frac{16}{33}$.

So $P=1-\frac{16}{33}=\frac{17}{33}$ - the same answer as above.

Answer: C.

Discussed at: http://gmatclub.com/forum/probability-96078.html

32. Six mobsters have arrived at the theater for the premiere of the film "Goodbuddies." One of the mobsters, Frankie, is an informer, and he's afraid that another member of his crew, Joey, is on to him. Frankie, wanting to keep Joey in his sights, insists upon standing behind Joey in line at the concession stand, though not necessarily right behind him. How many ways can the six arrange themselves in line such that Frankie's requirement is satisfied?
A. 6
B. 24
C. 120
D. 360
E. 720

Arrangement of 6=6!. In half of the cases Frankie will be behind Joey and in half of the cases Joey will be behind Frankie (as probability doesn't favor any of them). So, the needed arrangement is 6!/2=360.

Answer: D (360)

Discussed at: http://gmatclub.com/forum/permutation-combination-86147.html

33. In a room filled with 7 people, 4 people have exactly 1 sibling in the room and 3 people have exactly 2 siblings in the room. If two individuals are selected from the room at random, what is the probability that those two individuals are NOT siblings?
A. 5/21
B. 3/7
C. 4/7
D. 5/7
E. 16/21

As there are 4 people with exactly 1 sibling each: we have two pairs of siblings (1-2; 3-4).
As there are 3 people with exactly 2 siblings each: we have one triple of siblings (4-5-6).

Solution #1:

# of selections of 2 out of 7 - $C^2_7=21$;

# of selections of 2 people which are not siblings - $C^1_2*C^1_2$ (one from first pair of siblings*one from second pair of siblings)+$C^1_2*C^1_3$ (one from first pair of siblings*one from triple)+ $C1_2*C^1_3$(one from second pair of siblings*one from triple) $=4+6+6=16$.

$P=\frac{16}{21}$

Solution #2:

# of selections of 2 out of 7 - $C^2_7=21$;

# of selections of 2 siblings - $C^2_3+C^2_2+C^2_2=3+1+1=5$;

$P=1-\frac{5}{21}=\frac{16}{21}$.

Solution #3:
$P=2*\frac{3}{7}*\frac{4}{6}+2*\frac{2}{7}*\frac{2}{6}=\frac{4}{7}+\frac{4}{21}=\frac{16}{21}$.

Answer: E.

Discussed at: http://gmatclub.com/forum/mgmat-cat-picking-friends-87550.html

34. At a blind taste competition a contestant is offered 3 cups of each of the 3 samples of tea in a random arrangement of 9 marked cups. If each contestant tastes 4 different cups of tea, what is the probability that a contestant does not taste all of the samples?

A. $\frac{1}{12}$

B. $\frac{5}{14}$

C. $\frac{4}{9}$

D. $\frac{1}{2}$

E. $\frac{2}{3}$

"The probability that a contestant does not taste all of the samples" means that contestant tastes only 2 samples of tea (one sample is not possible as contestant tastes 4 cups>3 of each kind).

$$\frac{C_3^2*C_6^4}{C_9^4}=\frac{5}{14}.$$

C_3^2 - # of ways to choose which 2 samples will be tasted;
C_6^4 - # of ways to choose 4 cups out of 6 cups of two samples (2 samples*3 cups each = 6 cups);
C_9^4 - total # of ways to choose 4 cups out of 9.

Answer: B.

Another way:

Calculate the probability of opposite event and subtract this value from 1.

Opposite event is that contestant will taste ALL 3 samples, so contestant should taste 2 cups of one sample and 1 cup from each of 2 other samples (2-1-1).

C_3^1 - # of ways to choose the sample which will provide with 2 cups;
C_3^2 - # of ways to chose these 2 cups from the chosen sample;
C_3^1 - # of ways to chose 1 cup out of 3 from second sample;
C_3^1 - # of ways to chose 1 cup out of 3 from third sample;
C_9^4 - total # of ways to choose 4 cups out of 9.

$$P=1-\frac{C_3^1*C_3^2*C_3^1*C_3^1}{C_9^4}=1-\frac{9}{14}=\frac{5}{14}.$$

Answer: B.

Discussed at: http://gmatclub.com/forum/prob-gclub-diagonostics-86830.html

35. A man chooses an outfit from 3 different shirts, 2 different pairs of shoes, and 3 different pants. If he randomly selects 1 shirt, 1 pair of shoes, and 1 pair of pants each morning for 3 days, what is the probability that he wears the same pair of shoes each day, but that no other piece of clothing is repeated?
A (1/3)^6 *(1/2)^3
B (1/3)^6*(1/2)
C (1/3)^4
D (1/3)^2*(1/2)
E 5(1/3)^2

For the first day he can choose **any** outfit, $p=1$;
For the second day he must choose the same shoes as on the first day and different shirts and pants form the first day's, $p=\frac{1}{2}*\frac{2}{3}*\frac{2}{3}=\frac{2}{9}$;
For the third day he must choose the same shoes as on the first day and different shirts and pants from the first and second day's, $p=\frac{1}{2}*\frac{1}{3}*\frac{1}{3}=\frac{1}{18}$;

$$P=1*\frac{2}{9}*\frac{1}{18}=\frac{1}{81}=\frac{1}{3^4}$$

Answer: C - $\frac{1}{3^4}$

Discussed at: http://gmatclub.com/forum/probability-of-wearing-dress-91717.html

36. Anthony and Michael sit on the six-member board of directors for company X. If the board is to be split up into 2 three-person subcommittees, what percent of all the possible subcommittees that include Michael also include Anthony?
A. 20%
B. 30%
C. 40%
D. 50%
E. 60%

First approach:
Let's take the group with Michael: there is a place for two other members and one of them should be taken by Anthony, as there are total of 5 people left, hence there is probability of 2/5=40%.

Second approach:
Again in Michael's group 2 places are left, # of selections of 2 out of 5 5C2=10 - total # of outcomes.
Select Anthony - 1C1=1, select any third member out of 4 - 4C1=4, total # =1C1*4C1=4 - total # of winning outcomes.
P=# of winning outcomes/# of outcomes=4/10=40%

Third approach:
Michael's group:
Select Anthony as a second member out of 5 - 1/5 and any other as a third one out of 4 left 4/4, total=1/5*4/4=1/5;
Select any member but Anthony as second member out of 5 - 4/5 and Anthony as a third out of 4 left 1/4, total=4/5*1/4=1/5;
Sum=1/5+1/5=2/5=40%

Fourth approach:
Total # of splitting group of 6 into two groups of 3: 6C3*3C3/2!=10
# of groups with Michael and Anthony: 1C1*1C1*4C1=4
P=4/10=40%

Answer: C.

Discussed at: http://gmatclub.com/forum/combination-anthony-and-michael-sit-on-the-six-member-87081.html

37. Two couples and one single person are seated at random in a row of five chairs. What is the probability that neither of the couples sits together in adjacent chairs?
A 1/5
B. 1/4
C. 3/8
D. 2/5
E. 1/2

Let's find the opposite probability and subtract it from 1.

Opposite event that **neither of the couples sits together** is event that **at leas one couple sits together**. # of arrangements when at leas one couple sits together is sum of arrangements when EXACTLY 2 couples sit together and EXACTLY 1 couples sit together.

Couple A: A1, A2
Couple B: B1, B2
Single person: S

EXACTLY 2 couples sit together:
Consider each couple as one unit: **{A1A2}{B1B2}{S}**, # of arrangement would be: $3!*2!*2!=24$. 3! # of different arrangement of these 3 units, 2! arrangement of couple A (A1A2 or A2A1), 2! arrangement of couple B (B1B2 or B2B1).

EXACTLY 1 couples sit together:
Couple A sits together: **{A1A2}{B1}{B2}{S}**, # of arrangement would be: $4!*2!=48$. 4! # of different arrangement of these 4 units, 2! arrangement of couple A (A1A2 or A2A1). But these 48 arrangements will also include arrangements when 2 couples sit together, so total for couple A would be $48-24=24$;

The same for couple B: **{B1B2}{A1}{A2}{S}**, # of arrangement would be: $4!*2!=48$. Again these 48 arrangements will also include arrangements when 2 couples sit together, so total for couple B would be $48-24=24$;

$24+24=48$.

Finally we get the # of arrangements when at least one couple sits together is $24+48=72$.

Total # of arrangements of 5 people is $5!=120$, hence probability of an event that **at leas one couple sits together** would be $\frac{72}{120}=\frac{3}{5}$.

So probability of an event that **neither of the couples sits together** would be $1-\frac{3}{5}=\frac{2}{5}$

Answer: D.

Discussed at: http://gmatclub.com/forum/2-couples-and-a-single-person-probability-question-92400.html

38. As part of a game, four people each must secretly choose an integer between 1 and 4, inclusive.
What is the approximate likelihood that 2 people will choose same number?
What is the approximate likelihood that 3 people will choose same number?

When four people choose an integer between 1 and 4, inclusive 5 cases are possible:

A. All choose different numbers - {a,b,c,d};
B. Exactly 2 people choose same number and other 2 choose different numbers - {a,a,b,c};
C. 2 people choose same number and other 2 also choose same number - {a,a,b,b};
D. 3 people choose same number - {a,a,a,b};
E. All choose same number - {a,a,a,a}.

Some notes before solving:
As only these 5 cases are possible then the sum of their individual probabilities must be 1: $P(A)+P(B)+P(C)+P(D)+P(E)=1$

$$Probability=\frac{\#\ of\ favorable\ outcomes}{total\ \#\ of\ outcomes}$$

As each person has 4 options, integers from 1 to 4, inclusive, thus denominator, total # of outcomes would be 4^4 for all cases.

A. All choose different numbers - {a,b,c,d}:

$P(A)=\frac{4!}{4^4}=\frac{24}{256}$.

# of ways to "assign" four different objects (numbers 1, 2, 3, and 4) to 4 persons is 4!.

B. Exactly 2 people choose same number and other 2 choose different numbers - {a,a,b,c}:

$P(B)=\frac{C^2_4*4*P^2_3}{4^4}=\frac{144}{256}$.

C^2_4 - # of ways to choose which 2 persons will have the same number;
4 - # of ways to choose which number it will be;
P^2_3 - # of ways to choose 2 different numbers out of 3 left for 2 other persons when order matters;

C. 2 people choose same number and other 2 also choose same number - {a,a,b,b}:

$P(C)=\frac{C^2_4*\frac{4!}{2!2!}}{4^4}=\frac{36}{256}$.

C^2_4 - # of ways to choose which 2 numbers out of 4 will be used in {a,a,b,b};
$\frac{4!}{2!2!}$ - # of ways to "assign" 4 objects out of which 2 a's and 2 b's are identical to 4 persons;

D. 3 people choose same number - {a,a,a,b}:

$P(D)=\frac{C^3_4*4*3}{4^4}=\frac{48}{256}$.

C^3_4 - # of ways to choose which 3 persons out of 4 will have same number;
4 - # of ways to choose which number it will be;
3 - options for 4th person.

E. All choose same number - {a,a,a,a}:

$P(E)=\frac{4}{4^4}=\frac{4}{256}$.

4 - options for the number which will be the same.

Checking: $P(A)+P(B)+P(C)+P(D)+P(E)=\frac{24}{256}+\frac{144}{256}+\frac{36}{256}+\frac{48}{256}+\frac{4}{256}=1$.

Discussed at: http://gmatclub.com/forum/probability-simple-question-98684.html

39. Diana is going on a school trip along with her two brothers, Bruce and Clerk. The students are to be randomly assigned into 3 groups, with each group leaving at a different time. What is the probability that DIana leaves at the same time as AT LEAST on her bothers?
A. 1/27
B. 4/27
C. 5/27
D. 4/9
E. 5/9

Diana and her two brothers can be assigned to one of the 3 groups, so each has 3 choices, so total # of different assignments of Diana and her 2 brothers to 3 groups is $3*3*3=3^3=27$.

Now, "Diana leaves at the same time as AT LEAST one her brothers" means that Diana is in the same group as at least one her brothers.

Let's find the opposite probability of such event and subtract it from 1. Opposite probability would be the probability that **Diana is not in the group with any of her brothers**.

In how many ways we can assign Diana and her two brothers to 3 groups so that Diana is not in the group with any of her brothers? If Diana is in the first group, then each of her two brothers will have 2 choices (either the second group or the third) and thus can be assigned in $2*2=2^2=4$ ways to other two groups. As there are 3 groups, then total # of ways to assign Diana and her 2 brothers to these groups so that Diana is not in the group with any her bothers is $3*4=12$ (For each of Diana's choices her brother can be assigned in 4 ways, as Diana has 3 choices: first, second or the third group, then total $3*4=12$). So probability of this event is $\frac{12}{27}$.

Probability that Diana is in the same group as at least one her brothers would be $1-\frac{12}{27}=\frac{15}{27}=\frac{5}{9}$.

Answer: E.

Discussed at: http://gmatclub.com/forum/probability-qs-from-princeton-review-96989.html

40. Kate and David each have $10. Together they flip a coin 5 times. Every time the coin lands on heads, Kate gives David $1. Every time the coin lands on tails, David gives Kate $1. After the coin is flipped 5 times, what is the probability that Kate has more than $10 but less than $15?
A. 5/16
B. 15/32
C. 1/2
D. 21/32
E. 11/16

After 5 tries Kate to have more than initial sum of 10$ and less than 15$ must win 3 or 4 times (if she wins 2 or less times she'll have less than 10$ and if she wins 5 times she'll have 15$).

So the question becomes "what is the probability of getting 3 or 4 tails in 5 tries?".

$$P(t=3\ or\ t=4)=P(t=3)+P(t=4)=C_5^3*(\frac{1}{2})^5+C_5^4*(\frac{1}{2})^5=\frac{15}{32}$$

Answer: B.

To elaborate more:

If the probability of a certain event is p, then the probability of it occurring k times in n-time sequence is: $P=C_n^k*p^k*(1-p)^{n-k}$

For example for the case of getting 3 tails in 5 tries:
$n=5$ (5 tries);
$k=3$ (we want 3 tail);
$p=\frac{1}{2}$ (probability of tail is 1/2).

So, $P=C_n^k*p^k*(1-p)^{n-k}=C_5^3*(\frac{1}{2})^3*(1-\frac{1}{2})^{(5-3)}=C_5^3*(\frac{1}{2})^5$

OR: probability of scenario t-t-t-h-h is $(\frac{1}{2})^3*(\frac{1}{2})^2$, but t-t-t-h-h can occur in different ways:

t-t-t-h-h - first three tails and fourth and fifth heads;

h-h-t-t-t - first two heads and last three tails;
t-h-h-t-t - first tail, then two heads, then two tails;
...

Certain # of combinations. How many combinations are there? Basically we are looking at # of permutations of five letters t-t-t-h-h, which is $\frac{5!}{3!2!}$.

Hence $P=\frac{5!}{3!2!}*(\frac{1}{2})^5$.

Discussed at: http://gmatclub.com/forum/probability-97177.html